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Confidence intervals in temperature-based death time determination

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ABSTRACT

Marshall and Hoare's double exponential model with Henßge's parameters is a well known method for temperature based death time estimation. The authors give 95%-confidence intervals for their method. Since body cooling is a complex thermodynamical process, one has to take into account a potential bias of the estimator. This quantity measures the systematic error of the estimators underlying model. For confidence interval radius calculation a bias of 0 is presupposed, therefore the actual probability of the true death time value to lie in the 95%-confidence interval can be much lower than 95% in case of nonvanishing bias.

As in case of nonstandard conditions the confidence intervals have a probability of containing the true death time value which even in case of small corrective factor errors of $\Delta c = \pm 0.1$ can be substantially smaller than the 95% claimed, the paper presents a formula for confidence intervals which keep a 95% probability in case of error $\Delta c \leqslant \pm 0.1$.

1. Introduction

An elaborate version of this article is provided in the electronic supplementary material giving more detailed explanations and discussions.

Nearly every scientific quantitative estimation technique provides a 95%-confidence interval in addition to its estimation results. This interval is meant to be a measure of precision of the particular estimation. Its computation is performed under the tacit assumption of the estimation being correct, meaning that the deviation of the estimator value from the true value is merely the result of random variations of the estimation processes input data. This assumption excludes estimation errors which are caused by systematic model errors resulting in a so called bias. Usually the bias of an estimator $t^\wedge$ (e.g. the death time estimator based on MHH[1] (see [1])) with probability distribution $P_{t^\wedge}$ is defined as the deviation of the expectation value $E(t^\wedge)$ from the true value t (e.g. the true time of death) estimated. The paper presented investigates the impact of a non vanishing bias on the probability of a confidence interval computed for an estimation $t^\wedge$ with zero bias assumed.

Estimating the time of death in criminal cases is an important task in forensic medicine. A widely applied method for this purpose is rectal temperature based death time determination. There have been alternative approaches to use rectal temperature for death time backcalculation (see [2–4]) but still the most widely applied method is the approach MHH (see [5,6,1,7,8]). The method uses the well known double exponential model (see [9]) for the rectal temperature $T_R(t)$ – as a function of time t post mortem – which contains the ambient temperature T_A, the rectal temperature T_0 at time of death, the body weight m and a corrective factor c. The factor c was designed to cope with a great variety of so called non-standard conditions (clothing, body position, substrate, moving air,...) of the case investigated. Choosing incorrect values of T_A, T_0, m, c leads to an incorrect rectal cooling model curve and therefore to a bias b in death time determination. Since the variety of possible non-standard conditions of a cooling case is a potentially infinite set, the determination of the factor c is often performed by comparing the actual case to a list of cases in forensic medicine literature (e.g. [10]: Table 6, [7]: Table 6.7) for each of which the optimal corrective factor c was computed and listed. So errors in corrective factor determination have to be expected and to be coped with. Since each error in the corrective factor c leads to a bias b in death time determination, the 95%-confidence intervals are affected (see Fig. 1). As usage of MHH and its confidence intervals can lead to convictions in murder cases, the possible sources of 95%-confidence interval probability distortion should be carefully studied. Apart from empirical approaches (see [11–14]) only few studies were performed to investigate temperature based death time determination error on a theoretical basis (e.g. [15–17]).

We emphasize here that all results of this paper were yielded under the assumption that all biases in the application of MHH

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[1] MHH – post mortem rectal temperature cooling model of Marshall and Hoare with parameter definitions of Henßge

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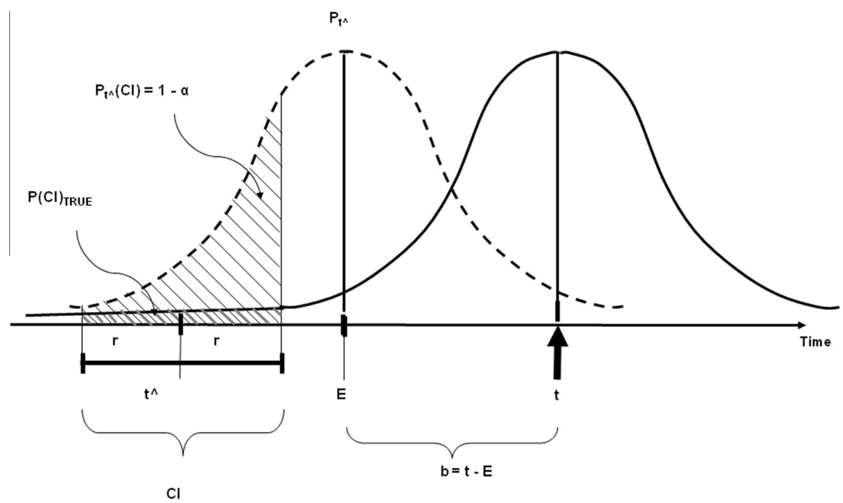

Fig. 1. Scenario of confidence interval probability computation: t = true death time value, $t^\wedge$ = estimated death time value with biased estimator, E = expectation value of biased estimator $t^\wedge$, r = confidence interval radius, b = bias of estimator $t^\wedge$, CI = confidence interval estimation based on biased death time estimation $t^\wedge$, drawn curve = pdf of hypothetically unbiased estimator, dashed curve = pdf $P_{t^\wedge}$ of biased real estimator t^.

time since death estimator stem from external sources such as erroneous corrective factor choice or error in temperature recording. We further assumed for our approach the standard deviations given in the MHH literature (e.g. [7,8]).

2. Methods and results

The study presented can be interpreted as a sort of theoretical consistency test of MHHs probability statements. Therefore the schemes of our computations present the methods and their outcome the result of the study. Since it seems to be rather inconvenient for the reader to artificially divide computation and outcome, we will present both in one section.

2.1. Confidence interval estimation in the death time estimation method of Marshall and Hoare and Henßge

In the articles (see e.g. [6,11]) establishing the temperature based death time estimation MHH, the authors computed $(1 - \alpha)$ – confidence intervals with $\alpha = 0.05$ (95%-confidence interval). In cases of non-standard conditions for each calibration case a corrective factor value c was chosen, which led to an optimal death time estimation. Since it can be difficult to choose an optimal corrective factor c in application cases, where the true death time is unknown, the authors of [11] computed the differences not only using the value c for each body but also with the neighboring values $c + 0.1$ and $c - 0.1$ and pooled the additional values in the difference histogram for confidence interval estimation as well.

2.2. Confidence interval estimation under bias

Let t be the true value of the time since death to be estimated by the random variable $t^\wedge$ estimated time since death in MHH called the estimator. Let further $E:= E(t^\wedge)$ be the expectation value and $s:= s(t^\wedge)$ be the standard deviation of the estimator $t^\wedge$. The estimator $t^\wedge$'s model bias $b:= b(t^\wedge)$ is the difference between the true death time value t and the expectation value E of the estimator:

$$b := t - E \tag{2.1}$$

Defining α as a small probability value (e.g. $\alpha = 0.05$ in case of 95%-confidence intervals) one yields the following formula (Eq. (2.2)) for the true probability $P(\mathrm{CI})$ of a $(1 - \alpha)$-confidence interval $\mathrm{CI} = [t^\wedge - r,\ t^\wedge + r]$ (e.g. $1 - \alpha = 0.95$ in case of a 95%-confidence interval CI) if the confidence interval radius r was computed under the erroneous assumption of zero bias $b = 0$. Assuming a Normal distribution with standard deviation s and cumulative probability distribution function Φ for the estimator $t^\wedge$, we can write formula (Eq. (2.2)) to quantify the true probability $P(\mathrm{CI})$ of the confidence interval CI (detailed derivation in Supplementary Material: Appendix A). The easy idea of the derivation is shown in Fig. 1:

$$P(\mathrm{CI}) = \Phi\left(\frac{b + r}{s}\right) - \Phi\left(\frac{b - r}{s}\right) \tag{2.2}$$

Equation (Eq. (2.2)) can numerically be solved for the CI-radius r, the bias b or the standard deviation s.

2.3. Confidence interval estimation in case of non-standard conditions in MHH

MHH instruct the user to apply corrective factors c in case of so called non-standard cooling conditions (e.g. [10]). By standard conditions the authors mean ambient conditions (clothing, air movement, body position, substrate,...) which are adequately similar to the ambient conditions of their model calibration experiments. Since the set of all possible environmental conditions is a vast (see e.g. [18]) for possible influences on thermodynamical processes) – in fact infinite – class, which has to be represented by the possible values of only one real model variable c, the authors of MHH published tables (e.g. [7,8,10]) of cases with independently known times of death, listed the ambient conditions for each case and added for each case a value c which was tuned to result in the correct death time if applied in MHH. Aside from a few general rules (e.g. in [8,19]) for choosing c, the determination of an appropriate corrective factor in real world case work is done essentially by browsing the aforementioned tables for a sufficiently similar case and taking its corrective factor c. Being the most reasonable procedure in this situation the approach nevertheless is prone to errors caused by choosing a wrong corrective factor and thereby introducing a bias into the confidence interval calculation. The authors give enlarged 95%-confidence interval radii r (e.g. in [8]) to cope with the problem (for the computation of r see Supplementary material: paragraph 2.1, procedure (N)) and state the enlarged radii to be robust against a corrective factor error of order $\Delta c = \pm 0.1$ (e.g. in [7] Table 6.9: "Die angegebenen Fehlerbreiten schließen eine Fehlschätzung des Korrekturfaktors von ±0.1 um den verwendeten Faktor ein."). We interpret this statement as:

(I) Even if the error Δc in the correction factor c is $\Delta c = \pm 0.1$ the true death time value t still lies in CI: $= [t_c - r,\ t_c + r]$ with 95% probability.

It is easy to derive the approximative formula (Eq. (2.3)) (derivation details in Supplementary Material: Appendix B) for the true probability $P(\mathrm{CI})$ of a $(1-\alpha)$-confidence interval CI, whose radius r was erroneously computed assuming a corrective factor value of c whereas $c - \Delta c$ respectively $c + \Delta c$ would had been the true value:

$$P(\mathrm{CI}) = \Phi\left(\frac{t\,\Delta c/c + r}{s}\right) - \Phi\left(\frac{t\,\Delta c/c - r}{s}\right) \tag{2.3}$$

The computations performed above offer the possibility to provide confidence intervals CI which fulfil claim (I): After performing a backcalculation yielding the death time estimator value t^ one assumes the order of magnitude of the true death time value t being approximated by the estimator values: $t \approx t$^. The value t and the maximum corrective factor deviation Δc permitted by the confidence interval CI, as well as the standard deviation s, which is associated to the actual reduced temperature value Q: $= (T_R - T_A)/(T_0 - T_A)$, are inserted in formula (Eq. (2.3)) and the formula is numerically solved for $r = r(t)$. As a demonstration example we look at a hypothetical case with a death time value of $t \approx t$^ = 20 h, a reduced temperature of $Q = 0.7$, a maximum corrective factor deviation $\Delta c = 0.1$ and a corrective factor value of $c = 1.0$. Numerically solving formula (Eq. (2.3)) for r yields a radius of $r = 4.2$ h for our 95% confidence interval instead of the radius $r = 2.8$ h recommended by the usual procedure. (Supplementary Material: Appendix D provides a more detailed interpretation of (I)).

3. Discussion

The results of temperature based death time determination as well as their associated confidence interval estimations can play crucial roles in murder investigations and homicide trials. Since the seventies of the last century the most established approach for temperature based death time determination is the method MHH. The article presented investigates the statistical consistency of the confidence interval estimation approach in MHH without – and under bias.

In Section 2.2 a general formula (Eq. (2.2)) is presented, quantifying the probability of a biased estimators confidence interval which erroneously was estimated under the assumption of zero bias.

Under nonstandard conditions MHH claims a certain kind of robustness (see [11]) of the confidence interval CI $= [t_c - r,\ t_c + r]$ against error in the corrective factor c of order $-0.1 \le \Delta c \le +0.1$: “Die angegebenen Fehlerbreiten schließen eine Fehlschätzung des Korrekturfaktors von ±0.1 um den verwendeten Faktor ein.” (Translation by the authors of the present article: “The error widths presented include a corrective factor estimation error of ±0.1.”). We tried to make this statement more explicit by outlining its claim (I). Claim (I) states that even in case of maximal correction factor error $\Delta c = \pm 0.1$ the confidence interval CI still contains the true time t of death with 95% probability. Formula (Eq. (2.3)) gives MHH-users the possibility to enlarge the confidence interval radius r to actually fulfill claim (I).

An example may help to illustrate the course of $P(\mathrm{CI})$ and of an enhanced CI radius r, which in this version fulfils claim (I), with rising time since death t. In the book [7] Table 6.8 Fall 25 the case of a body of $m = 100.5$ kg cooling in standing water of $T_A = 19.8$ °C is presented. The correction factor $c = 0.5$ is applied and the precision of backcalculation is stated as usual in non-standard cases. Applying our results in this case makes it possible to compute the actual 95%-confidence interval probability P(CI) as a function of real time t since death (see Fig. 2) in the worst case scenario where the real corrective factor was not $c = 0.5$ but $c + \Delta c = 0.6$ or alternatively $c - \Delta c = 0.4$. Fig. 3 shows the (I)-enhanced confidence interval radius r as a function of real time since death. In both diagrams we sketched the limits of the time intervals corresponding to $1.0 \geqslant Q > 0.5$ ($s = 1.3$ h, $r = 2.8$ h), $0.5 \geqslant Q > 0.3$ ($s = 2.2$ h, $r = 4.5$ h) and $0.3 \geqslant Q > 0.2$ ($s = 3.4$ h, $r = 7.0$ h) and traced the relevant parts of the graphs fat. In case of $s = 1.3$ h/$s = 2.2$ h/$s = 3.4$ h the minimum $P(\mathrm{CI})$ is down to $P(\mathrm{CI}) = 0.66/P(\mathrm{CI}) = 0.67/P(\mathrm{CI}) = 0.78$ (see Fig. 2) whereas the maximum (I)-enhanced radii are $r = 4.3$ h/$r = 7.1$ h/$r = 10.2$ h (see Fig. 3).

We can generally note that higher corrective factor values and higher values of the confidence interval radius yield confidence intervals more robust against hidden bias.

In sum the results presented should be taken as warning signs not to forget the possibility of bias when stating 95%-confidence

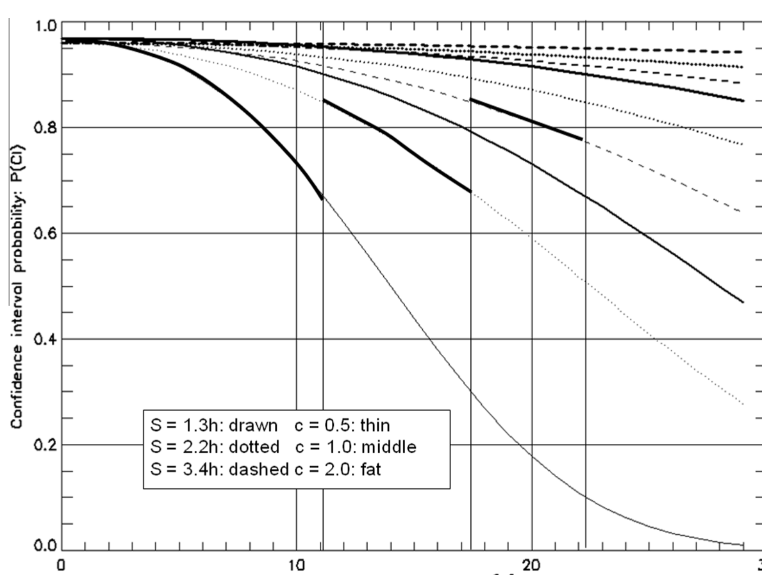

Fig. 2. Probability $P(\mathrm{CI})$ of the ‘global’ 95%-confidence interval CI in case of maximal corrective factor choosing error $\Delta c = \pm 0.1$ for case 25 from [7] Table 6.8 (fat traced graphs parts) with parameter values $c = 0.5$, $m = 100.5$ kg in standing water of $T_A = 19.8$ °C.

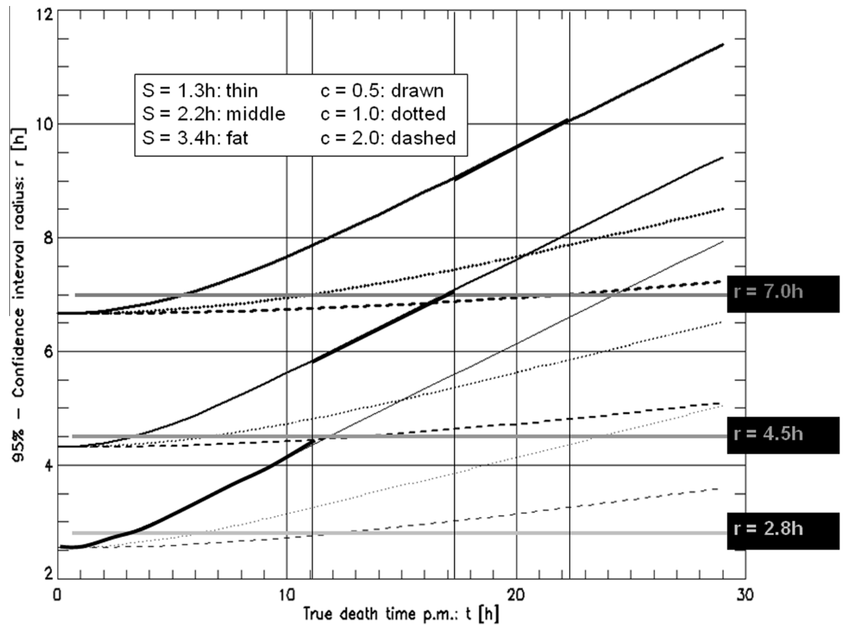

Fig. 3. (I1)-Enhanced 95%-confidence interval radius $r = r(t)$ as a function of true time t since death in case of maximal corrective factor error $\Delta c = \pm 0.1$ for case 25 from [7] Table 6.8 (fat traced graph parts) with parameter values $c = 0.5$, $m = 100.5$ kg in standing water of $T_A = 19.8$ °C.

intervals in expertises and in court. Additionally the experts should bear in mind the fact that error in corrective factor choice is not avoidable in any case (see e.g. [1]) and that assumed 95% intervals can have dramatical loss of probability as a consequence.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.legalmed.2014.08.002.

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